Math 216 - Exam 2 Solutions

Fall 2008 - Hartlaub

1. (a) Use Kendall's correlation statistic to test

$H_0: \tau = 0$ vs $H_1: \tau > 0$

$$K = \sum_{i=1}^{16-1} \sum_{j=i+1}^{16} Q((x_i, y_i), (x_j, y_j))$$

$= \#$ of concordant pairs $- \#$ of discordant pairs

$= 68 - 47$

$= 21$

$p\text{-value} = P(K \geq 21) > .175$ (Table A.30)

Thus, we cannot reject H_0 (at any reasonable α level, say $\alpha = .05$). Head circumference and height are not significantly associated.

(b) Use Spearman's rank correlation

$$r_s = 1 - \frac{6 \sum_{i=1}^{16} D_i^2}{16(16^2-1)} \underset{MTB}{=} 0.188$$

Table A.31 $\Rightarrow$ $p\text{-value} = P(r_s > .188) > .10$.

r_s agrees with our conclusion in part a. r_s is positive but not large enough to be significantly greater than zero.

Pearson's correlation: $r = .158$, with a p-value of $.559$.

Pearson's statistic also agrees with the conclusion provided in part a.

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2. (a) $X_{ij} = \theta + \tau_j + e_{ij}$ $\quad i = 1, \ldots, 10$, $j = 1, 2, 3$,

where X_{ij} = r_s for psychologist i in treatment group j

θ = overall median

τ_j = group (or treatment) effect

e_{ij} = random error term from a continuous dist with median 0.

We want to test $H_0: [\tau_1 = \tau_2 = \tau_3]$ vs H_1: [not all τ's are equal]

Kruskal-Wallis Test Statistic $\underset{MTB}{=}$ $H = 5.82$

p-value = 0.054 (adjusted for ties)

Conclusion: At the $\alpha = .05$ level, we cannot reject H_0, since $.054 > .05$. However, the p-value is very close to the significance level. We do <u>not</u> have evidence that the measure of agreement differs significantly for these 3 groups. NOTE: We would make a different conclusion at the $\alpha = .06$ level.

(b) No, the alternative in part a is a general alternative, not an ordered alternative.

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2. (c) Test $H_0: [\tau_{61} = \tau_{62} = \tau_{63}]$
vs $H_1: [\tau_{63} \le \tau_{62} \le \tau_{61}$, with at least one $<]$

$$J = \sum_{u=1}^{v-1} \sum_{v=2}^{3} U_{uv} = U_{12} + U_{13} + U_{23}$$

$$= U_{63,62} + U_{63,61} + U_{62,61}$$

$$= 48 + 74.5 + 80$$

$$= 202.5$$

$$J^* = \frac{202.5 - \left[\frac{30^2 - (10^2 + 10^2 + 10^2)}{4}\right]}{\left\{\left[\frac{30^2(2\times 30+3) - 10^2(20+3) + 10^2(20+3) + 10^2(20+3)}{72}\right]\right\}^{1/2}}$$

$$= \frac{202.5 - 150}{26.2996}$$

$$= 1.9962$$

$$p\text{-value} = P(Z \ge 1.9962) = .023$$

Since $.023 < .05$, we reject H_0 and conclude that more information significantly decreases the closeness of agreement.

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#2 (d) No, each group is given some information (a treatment) to form opinions. Thus, we are comparing 3 treatment groups and we do not have a control group.

(e) The most informed group is group 3, so our contrast of interest is

$$\theta = \tau_3 - \tfrac{1}{2}(\tau_1 + \tau_2)$$

Estimate this contrast using $\hat{\theta} = \bar{\Delta}_3 - \tfrac{1}{2}\bar{\Delta}_1 - \tfrac{1}{2}\bar{\Delta}_2$

#3.

a. The state variable is a blocking variable (that is supposed to be ignored) i.e. we are not interested in comparing states. We are interested in comparing ratios over time.

$$\underbrace{X_{ij}}_{\text{ratios}} = \theta + \underbrace{\beta_i}_{\substack{\text{Block}\\\text{(state)}}} + \underbrace{\tau_j}_{\substack{\text{Treatment}\\\text{(year)}}} + \underbrace{e_{ij}}_{\text{error}} \qquad \begin{matrix} i=1,\dots,5 \\ j=1,\dots,6 \end{matrix}$$

$H_0: [\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6]$

vs $H_1: [\tau_1 \le \tau_2 \le \tau_3 \le \tau_4 \le \tau_5 \le \tau_6$, with at least one $<$]

Page's Test Statistic $= L = \sum_{j=1}^{6} j R_j$

$= 1(19) + 2(24) + 3(16) + 4(11) + 5(17) + 6(18)$

$= 352$

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#3 a. From Table A.23 (with $k=6$ and $n=5$, p. 705)
$\ell_{.0492} = 397$ or p-value $= P(L \geq 352) > .1065$

Since $352 < 397$ (or p-value $> .05$) we cannot reject H_0. These ratios do not significantly increase over time.

b. $X_{ij} = \theta + \beta_i + \tau_j + e_{ij}$

X_{ij}: ratios; β_i: Block (state); τ_j: Treatment (Year); e_{ij}: error

H_0: $[\tau_1 = \tau_2 = \tau_3 = \tau_4 = \tau_5 = \tau_6]$

H_1: [not all τ's are equal]

Friedman's test statistic: $S = 5.11$
p-value $= .402$

Since $.402 > .05$, we cannot reject H_0. We do not have statistically significant evidence of a difference in the ratios over time.

c) Model (b) is preferred because there is no reason provided for suspecting an increasing trend over time

d) No, since H_0 is not rejected, there is no reason to use multiple comparisons in this situation.